

What does a Circle Look like?

Which of these is a circle?



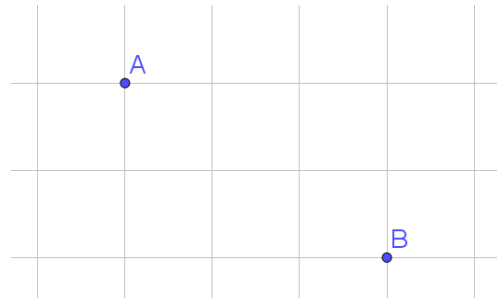
Answer: Both of them

We've come to think of a circle as a round thing. A more precise way to think about a circle is that it's the set of points that are within a certain distance (radius) away from a fixed point (centre). We get different-looking shapes if we change the definition of *distance*. The red circle is more famous because of our usual *as the crow flies* definition of distance. This definition isn't always relevant:

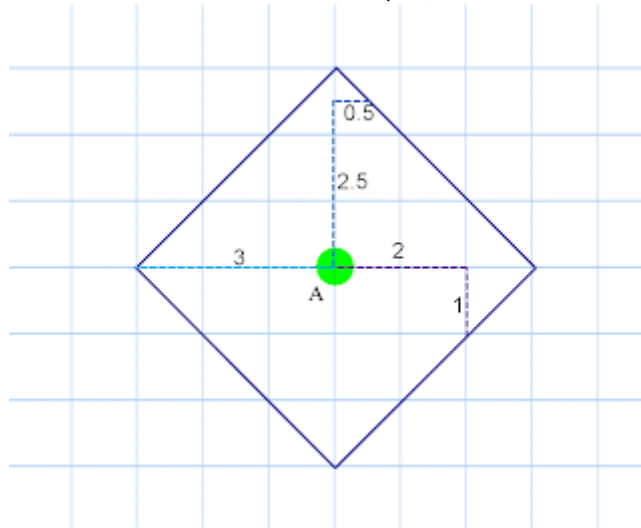


Imagine you are a taxi driver in Manhattan (a section of which is in the map above). The red notion of distance is irrelevant because you can't drive through buildings. The blue notion is more meaningful, and suggests a definition of *Taxicab distance* between 2 points as: *East to West difference + North to South difference*. This is the shortest distance that needs to be driven to get from A to B. (Note that there are many routes possible to achieve this shortest distance, 2 of which are displayed in blue on the map).

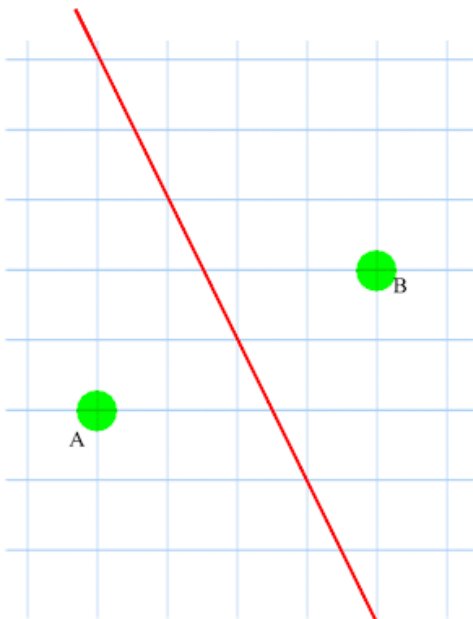
For example, the Taxicab distance between the two blue points, A and B, on this diagram is 5 (2 down and 3 across)



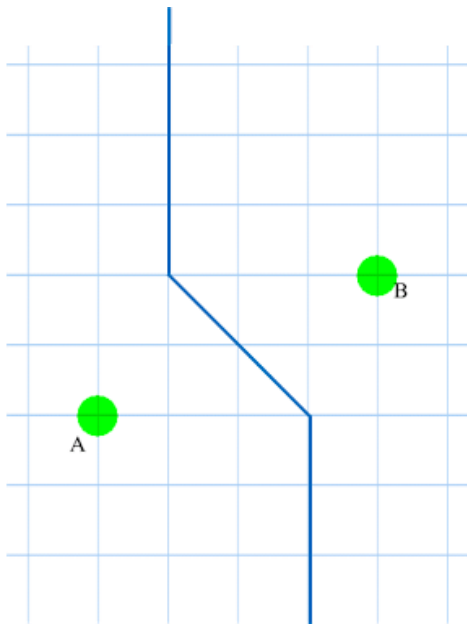
With this Taxicab distance definition, we get a geometry that looks different. For example, here is a circle with radius of 3:



Other things look different too:



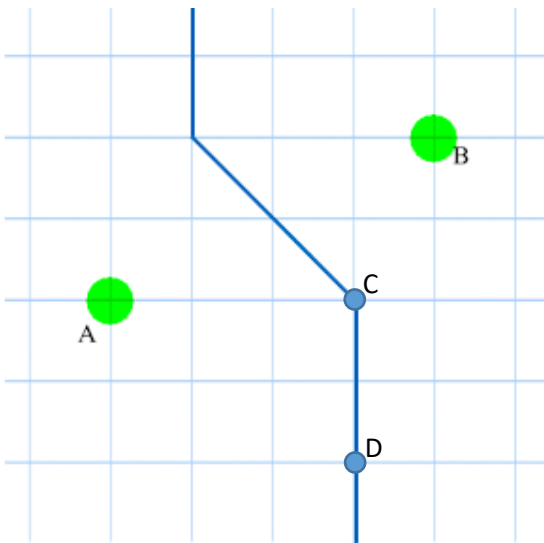
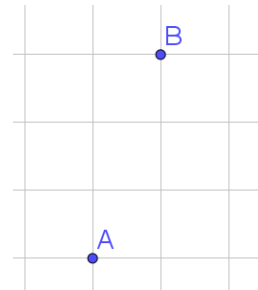
Set of points that are the same 'crow flies' distance from A as they are from B



Set of points that are the same 'Taxicab' distance from A as they are from B

What does a Circle Look like: Comprehension Questions:

- Which of the following is the best definition of a circle:
 - A set of points whose boundary (the circumference) consists of points that are an equal distance from a fixed point.
 - A round shape
 - A polygon with no straight edges.
- What is the Taxicab distance between the points A and B in the picture on the right?



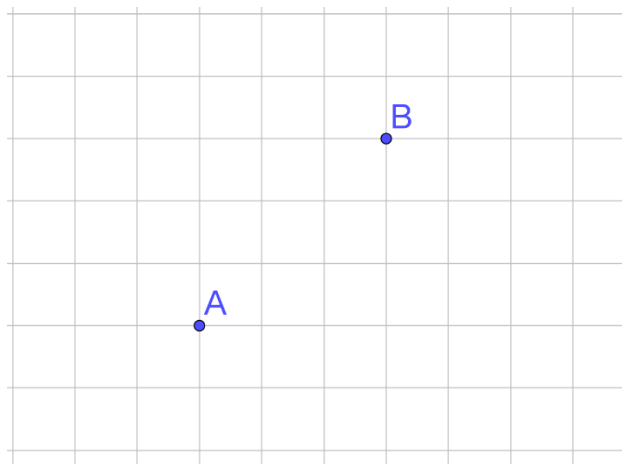
Set of points that are the same 'Taxicab' distance from A as they are from B

- In the article, I claim that all the points on the blue line on the picture on the left are the same Taxicab distance from A as they are from B.

Work out the following Taxicab distance, in order to test the claim:

- A to C
- B to C
- A to D
- B to D

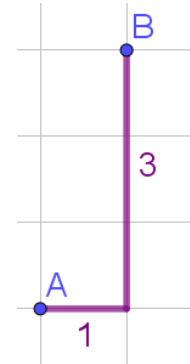
- Find as many points as you can on this grid that are the same Taxicab distance from A as they are from B



What does a Circle Look like: Comprehension Answers:

1. A) A circle is a set of points whose boundary (the circumference) consists of points that are an equal distance from a fixed point.

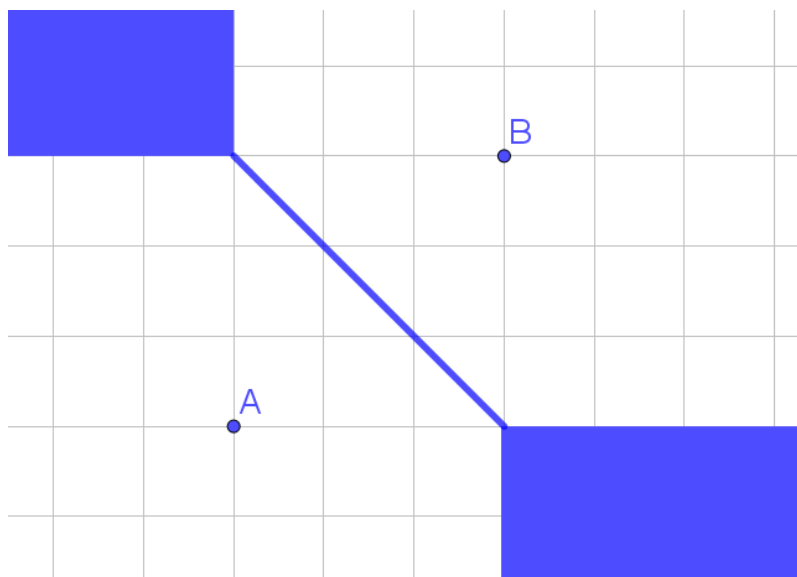
2. The Taxicab distance of A to B is 4, as this diagram shows:



- 3.

i) A to C and ii) B to C both have Taxicab distance of 3.
 iii) A to D and iv) B to D both have Taxicab distance of 5.

4. Here is a picture of points that are the same distance from A as they are from B. (Amazingly, the set of points is a 2-dimensional region, not just a line. I found this surprising when I first encountered it.)



(If we had a much larger piece of paper, the entire top left-hand and bottom right-hand regions would be coloured blue).