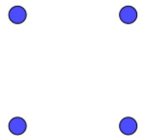
2-colourings

1. Draw 4 dots in square shape.



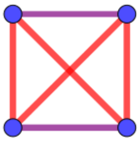
   Take two different colours (a pen and a pencil would be fine).

Rules of the game: You must connect each dot to each other dot, in a straight line, with one of your two colours.

Aim of the game: Try to avoid a *monochromatic triangle* (that is, a triangle, with dots at the *vertices* (corners), whose 3 sides are the same colour). If you manage this, you win.

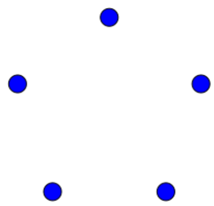
Did you win?

Here's one way of doing it:

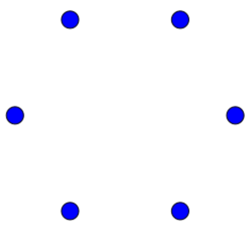


(Note that even though you might claim to see 2 red triangles, here, they're ok, because we're only interested in the triangles formed with vertices on the blue dots.)

2. Now, can you play the same game and win, starting with 5 dots, not 4?

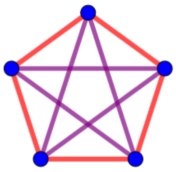


3. Now, can you play the same game and win, starting with 6 dots?



2-colourings - Answers and Further Thoughts

2. It is possible to win the 5 dot game. Here's an example:

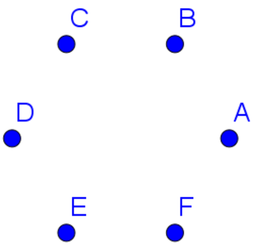


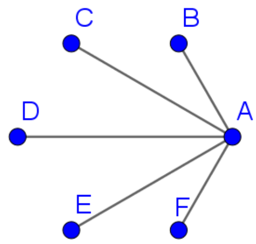
3. Please accept my apologies if you tried for a very long time to win the 6 dot game. It is impossible. Here's why:

Suppose our coloured pencils are red and purple.

Let's give each dot a name (A to F) and let's think about the lines from point A

There are 5 of these.

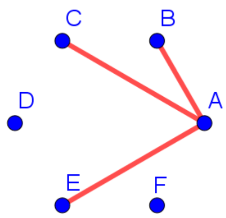


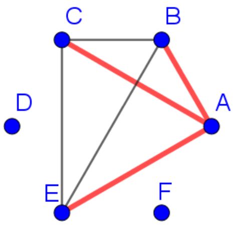


So there must be at least 3 of these lines coloured in the same colour (If not, then we would have only coloured at most 4 out of the 5 lines, which is against the rules).

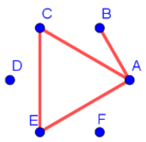
Suppose, then, that lines from A to B, C, and E are coloured, let's say, red.

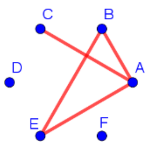
Let's now consider the vertices involving B, C and E. (black lines in the picture on the right)

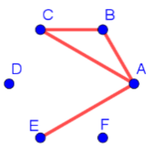




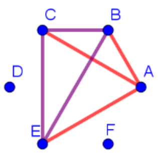
If any of these are coloured red, then they would form a red triangle:







So they must all be purple, but then we get a purple triangle:



Therefore it's impossible to avoid a monochromatic triangle.

Well done if you understood this proof. We have proved what's sometimes called the *Theorem on friends and strangers*, which states that: In any party of six people either at least three of them are (pairwise) mutual strangers or at least three of them are (pairwise) mutual acquaintances.

To find out more, look up *Ramsey Numbers* (This task has been about one particular Ramsey number, which is called R(3,3). We've proved that R(3,3) = 6.) There are lots of *Ramsey Numbers* that no one in the world knows, such as R(5,5).